Qualitative and Quantitative Approach to Ordinary Differential Equations through Mathematical Software

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Abstract

The implementation of qualitative and quantitative methods in Ordinary Differential Equations (ODEs) requires the use of mathematical software to achieve a modern approach with effectiveness in geometric and numerical analysis. The analysis of existence, uniqueness, and stability of differential equations is a difficult subject to address and understand without the necessary tools to understand the concepts involved and to visualize them graphically, geometrically, and numerically. This work was carried out with the objective of analyzing the solution of mathematical problems related to first-order ODEs. The Maple software was used, due to its great symbolic capacity, with an interface that makes it easy to analyze, explore, visualize and solve mathematical problems. First-order ODEs are analyzed and solved with Maple, considering existence, uniqueness, and stability. Also, a qualitative approach is discussed, obtaining qualitative information about the solutions directly from the equation, without the use of a formula for the solution. Worksheets have been built and developed in Maple containing the solution to some problems posed in the literature. Graphic and numerical representations are obtained that help carry out a convenient analysis and interpretation of the problems posed.

Keywords: Ordinary Differential Equations, Mathematical Software, Maple, Geometric and Numerical Analysis.

1 Introduction

The traditional introduction of Ordinary Differential Equations (ODEs) courses given by Zill [1] has concentrated in higher education a repertoire of techniques to find solution formulas for various classes of ODEs.

Typically, the result has been the application of formulaic techniques without a serious qualitative understanding of fundamental aspects of the topic, such as stability, asymptotic analysis, and parameter dependence. These fundamental ideas are difficult to understand because they have a lot of geometric content and involve a lot of calculations. Modern mathematical software systems can help overcome these difficulties. Collantes *et al*. [2-5] obtain their results with the help of Maple. Belyaeva *et al*. [6] constructed an algorithm and a program developed in the Maple environment in order to solve the fourth-order differential equations. For the use of qualitative and quantitative methods in ODEs, referring to its theory and applications, it is very important to have a powerful mathematical software being accessible to students and researchers. A modern system is the mathematical software Maple, effective in geometric and numerical analysis. Therefore, it is essential, first of all, to know its basic commands, as well as their use to perform calculations, graphs, derive and integrate functions. With this, a minimum level of competence will be achieved to allow the use of Maple in various ODEs topics.

In chapter 3: *Doing Mathematics with Maple*, Coombes *et al*. [7] present an introduction to the mathematical software Maple, moreover, in chapter 5: *Solutions of Differential Equations*, and chapter 6: *A Qualitative Approach to Differential Equations*, they present an analysis of the ODEs solutions and its qualitative approach, proposing sets of exercises and problems called *Problem Set A*: *Practice with Maple* and *Problem Set B*: *First Order Equations*, the same ones that are solved in Maple worksheets in this research.

2 Related works

Over the years, papers have been published on research projects using software for the analysis of differential equations. The works are discussed below in chronological order.

The need to solve large stiff and nonstiff ordinary differential equation systems (initial value problems) has led to considerable software. The growing number of good general purpose initial value solvers has fueled discussions on the idea of a systematized collection of such solvers, called ODE-PACK. These solvers are described briefly by Hindmarsh [8], and their capabilities are illustrated with an example problem arising from a system of PDE's modeling atmospheric kinetics-transport.

Neves and Thompson [9] reviewed the theoretical basis for the numerical solution of a general class of functional differential equations. They discussed a software package for the solution of differential equations with state-dependent delays. Examples are presented which demonstrate the manner in which the software takes into account the pertinent theoretical characteristics of functional differential equations.

Hereman [10] presented a survey of symbolic programs for the determination of Lie symmetry groups of systems of differential equations. The purpose, methods and algorithms of symmetry analysis are briefly outlined. Examples illustrate the use of the software.

Kunkel *et al*. [11] described the new software package GELDA for the numerical solution of linear differential-algebraic equations with variable coefficients. They give a brief survey of the theoretical analysis of linear differential-algebraic equations with variable coefficients and discuss the algorithms used in GELDA. Furthermore, they include a series of numerical examples as well as comparisons with results from other codes, as far as this is possible.

Where engineering math is concerned, computers haven't always made the going much easier. In fact, using computers to solve differential equations, particularly nonlinear or partial types, has often required tedious and time-consuming user intervention, as complex numerical tools. Recently, new solvers and other features have been introduced or incorporated into mathematics software to automate more of these steps. Meanwhile, developments in symbolic computation and the use of "packed arrays" of compressed data sets, new algorithms and programming libraries have brought more speed and power to mathematics software. Tackling the thorniest differential equations are solvers incorporated into Waterloo Maple Inc.'s. With Maple, users can instead write the expression "as is," and the software will sort it automatically until the solver kicks in. According to Shanley [12], these solvers have tried to reconcile the symbolic and the numerical world.

A vast collection of mathematical software, representing a significant source of mathematical expertise, is available now for use by scientists and engineers in their efforts for modelling the evolution of real systems. Petcu [13] discuss the state-of-the art in the field of software for ordinary differential equations (ODEs) and several issues which are of importance both from the point of view of software design and software or method evaluation.

The hyperbolic tangent (tanh) method is a powerful technique to symbolically compute traveling waves solutions of one-dimensional nonlinear wave and evolution equations. In particular, the method is well suited for problems where dispersion, convection, and reaction diffusion phenomena play an important role. Hereman and Malfliet [14] outlined this technique for the computation of closed form tanh-solutions for nonlinear partial differential equations and ordinary differential equations. Basic examples illustrate the key steps of the method. A brief survey is given of symbolic software packages that compute traveling wave solutions in closed form.

Cheviakov [15] presented a developed Maple-based “GeM” software package for automated symmetry and conservation law analysis of systems of partial and ordinary differential equations (DE). The package contains a collection of powerful easy-to-use routines for mathematicians and applied researchers. The “GeM” package is being successfully used in ongoing research. Run examples include classical and new results.

Lipsman *et al*. [16] have experimented at the University of Maryland over the last years with the use of several problem-solving environments (PSEs) to enhance the teaching and enrich the syllabus of the sophomore-level basic differential equations course for science and engineering majors. They have developed Educational materials for teaching this course in all of the “big three” PSEs: MATLAB (sometimes with Simulink also), Maple, and Mathematica, informats varying from small computer labs to large lecture courses. In this paper they revealed what they have learned about realistic goals for a course at this level, in terms of the software and the mathematics. Finally, they gave some assessment of the relative advantages and disadvantages of the three PSEs for a sophomore-level DE course.

Carneiro *et al*. [17] used specific softwares for a new approach for teaching ordinary and partial differential equations, in the field of heat transfer and fluid mechanics. The main objective of their paper was to enhance learning effectiveness of Numerical Methods in the post-graduate course of Polymers Engineering at the University of Minho. This degree takes place into two different environments: at the university campus and at the industrial field. Different commercial codes were used, namely EXCEL, MATLAB, and FLUENT, as well as two tools developed in house at University of Minho: CoNum and a graphics application PDE v.1. Lectures were based on videoconferencing and other web utilities. The teaching methodology presented and discussed in this article was well received and accepted by the post-graduate students, motivating teachers to improve their teaching/learning strategies.

Soetaert *et al*. [18] presented the R package **deSolve** to solve initial value problems (IVP) written as ordinary differential equations (ODE), differential algebraic equations (DAE) of index 0 or 1 and partial differential equations (PDE), the latter solved using the method of lines approach. The differential equations can be represented in R code or as compiled code. In the latter case, R is used as a tool to trigger the integration and post-process the results, which facilitates model development and application, whilst the compiled code significantly increases simulation speed. In this paper, their objectives are threefold: (1) to demonstrate the potential of using R for dynamic modeling, (2) to highlight typical uses of the different methods implemented and (3) to compare the performance of models specified in R code and in compiled code for a number of test cases.

Ahmed [19] presented the route used for solving differential equations for the engineering applications. Usually students of the Faculty of Engineering must enroll in a course of Differential Equations and Engineering Applications as a prerequisite for the subsequent stages of their study. Mainly, one of the objectives of this course is that the students practice MATLAB software package during the course. The aim of using this software is to solve differential equations individually and as a system of equations in parallel with analytical mathematics trends. In general, mathematical models of the engineering systems like mechanical, thermal, electrical, chemical and civil are modeled and solve to predict the behavior of the system under different conditions. The paper presents the technique which is used to solve DE using MATLAB. The main code that utilized and presented is MATLAB/ode45 to enable the students solving initial value DE and experience the response of the engineering systems for different applied conditions. Moreover, both advantages and disadvantages are presented especially the student mostly face in solving system of DE using ode45 code.

Latifi *et al*. [20] studied differential equations using important mathematical concepts like function, derivative, integral, etc. They used Geogebra as a dynamic mathematical software uniting geometry, algebra and differential calculus. There are two objectives in this study. The first objective is to examine the impact of the use of Geogebra on the students’ understanding of differential equations.

Semenova *et al*. [21] considered the use of the simulation to solve differential equations in MatLab/Simulink. The MatLab software package is designed to provide analytical and numerical solutions for various mathematical problems and simulate complex technical objects and systems. The Simulink app is one of the tools within the MatLab package. It has major structured, object-oriented, and visual programming capabilities. The system can solve linear algebra problems, integral and differential equations, and perform Laplace and Fourier transformations. They reviewed both the simplest equations (a first-order DE) and more complex linear and non-linear DE of the first and second orders.

3 Mathematical and Computational Foundations

For many ODEs, Maple's *dsolve* command produces its general solution, or the specific solution to an associated initial value problem.

3.1 Symbolic solutions

Consider the first order ordinary differential equation

$$\frac{dy}{dx} = f(x, y). \tag{1}$$

A solution to this equation is a function $y(x)$ of the independent variable x that satisfies $y'(x) = f(x, y(x))$. It is sometimes possible to find a formula for the solutions of (1); we call such a formula a symbolic solution. In Maple, the command that finds symbolic solutions is *dsolve*. To find a symbolic solution to the ordinary differential equation (1), write and execute

>dsolve(diff(y(x),x)=f(x,y(x)),y(x)). (2)

Maple produces the answer in terms of an arbitrary constant C1. For example, consider the ordinary differential equation $\frac{dy}{dx} = x + y$.

You can solve this equation in Maple by writing and executing

>dsolve(diff(y(x),x)=x+y(x),y(x)). (3)

The output of this command is $y(x) = -x - 1 + \mathrm{C1\_}e^{x}$. Specific solutions can be obtained by choosing specific values for C1. For example, the solution satisfying a given initial condition can be obtained by choosing C1 appropriately. This value of C1 can be found by imposing the initial condition on the general solution and solving for C1.

Alternatively, you can specify the initial condition as the ordinary differential equation when using *dsolve*.

To solve the initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0 \tag{4}$$

it is written and executed

>dsolve({diff(y(x),x)=f(x,y(x)),y(x0)=y0},y(x)). (5)

The *dsolve* command is robust, it can solve many ODEs. In fact, it can solve most ODEs that can be solved with standard solution methods. However, there are many other equations, some of which can be solved by more advanced solution methods that Maple cannot solve.

The existence, uniqueness and stability of ODEs solutions are discussed below, topics that are fundamental in the theory and application of ODEs. An understanding of them helps to interpret and use the results produced by Maple.

3.2 Existence and uniqueness

The fundamental existence and uniqueness theorems for ODEs, as is seen in Boyce & DiPrima [22], guarantee that every initial condition $y(x_0) = y_0$ leads to a single solution *close* to x_0, assuming that the right side of the ordinary differential equation (1) is an “appropriate” function (to be specific, assuming that f and $\partial f/\partial y$ are continuous functions).

Graphically, these theorems say that a solution curve exists through every point, and that solution curves cannot intersect. Thus, initial value problems have exactly one solution, but, since there are an infinite number of initial conditions, ODEs have an infinite number of solutions. This principle is implicit in the results obtained with *dsolve*.

When an initial condition is not specified the solution depends on an arbitrary constant; when an initial condition is specified, the solution is completely determined. It is important to remember that the existence and uniqueness theorems only guarantee the existence of a solution near the initial point x_0.

3.3 Stability

In addition to existence and uniqueness, sensitivity to the initial value of the solution of an initial value problem is a fundamental issue in the theory and application of ODEs.

Often, one is primarily interested in positive values of x, just like when x corresponds to the time in a physical problem. If an initial value problem is to predict the future of a physical system, the solution for positive x should be regularly insensitive to the initial value, this means that small changes in the initial value should lead to small changes in the solution for the positive time.

To see the importance of this principle, note that for a physical system the initial value y_0 is typically not known exactly, but it is found by measurements. When y_0 is measured, generally only an approximate value $\tilde{y}_0$ is obtained. Then if $\tilde{y}(x)$ is the solution corresponding to $\tilde{y}_0$, and the solution is very sensitive to the initial value, $\tilde{y}(x)$ will have little relation to the current state $y(x)$ of the system by increasing x.

An ordinary differential equation whose solution is regularly insensitive to the initial value when increasing x is called stable, however, when the solution is very sensitive to the initial value when increasing x, the ordinary differential equation is called unstable. As we have seen, if an equation is unstable and is used over a long-time interval, a small error in the initial value can subsequently cause a large error. Caution should be taken when using an unstable ordinary differential equation to model a physical problem.

Stability has been considered as it increases x, that is, stability to the right. Stability on the left can also be considered. There are equations that are stable both to the left and to the right and there are others that are unstable both to the left and to the right. The following theorem, which is stated without proof, is often useful in stability evaluation.

Theorem 1. Suppose that $f(x,y)$ has continuous first-order partial derivatives in the vertical region $R=\left\{(x,y):\ x_0\le x\le x_1,\ -\infty<y<\infty\right\}$, and suppose that there exist numbers K and L such that $K\le\dfrac{\partial f}{\partial y}(x,y)\le L,\ \ \forall(x,y)\in R.$

If $y(x)$ and $\tilde{y}(x)$ are solutions of $dy/dx=f(x,y)$ over the interval $x_0\le x\le x_1$ with initial values $y(x_0)=y_0$ and $\tilde{y}(x_0)=\tilde{y}_0$, respectively, then

$$\left|y_0-\tilde{y}_0\right|e^{K(x-x_0)}\le\left|y(x)-\tilde{y}(x)\right|\le\left|y_0-\tilde{y}_0\right|e^{L(x-x_0)},\quad \forall x_0\le x\le x_1. \qquad (6)$$

If $L\le 0,$ then the right-hand inequality in (6) shows that

$$\left|y(x)-\tilde{y}(x)\right|\le\left|y_0-\tilde{y}_0\right|,\ \ \forall x_0\le x\le x_1, \qquad (7)$$

In this way, the solutions differ by no more than the difference in the initial values, and the differential equation is stable. Furthermore, if $L>0,$ but not so big, and x_1-x_0 is not so big, then

$$\left|y(x)-\tilde{y}(x)\right|\le M\left|y_0-\tilde{y}_0\right|,\quad \forall x_0\le x\le x_1, \qquad (8)$$

where $M = e^{L(x_1 - x_0)}$ is a constant of moderate size. In this way, the equation is only slightly sensitive to changes in the initial value, and the equation is only slightly unstable. On the other hand, if $K > 0,$ then the left-hand inequality in (6) shows that the solution is sensitive to changes in the initial value, especially over long intervals.

We can briefly summarize these results by saying that if

$$\partial f \,/\, \partial y \le 0, \tag{9}$$

then the differential equation is stable, but if

$$\partial f \,/\, \partial y > 0, \tag{10}$$

then the equation is unstable.

The right-hand inequality in (6) of theorem is an example of *continuous dependence*, which shows that the solution depends continuously on the initial value.

3.4 Qualitative approach to an Ordinary Differential Equation: Direction Field

Consider the general first-order ordinary differential equation

$$\frac{dx}{dt} = f(t, x) \tag{11}$$

Qualitative information can be obtained about the solutions $x(t)$ observing (11) geometrically. Specifically, this information can be obtained from the direction field of (11). The direction field is obtained by drawing through each point (t, x) in the plane (t, x) a small line segment with a slope $f(t, x)$. Solutions, or integral curves of (11) have the property that at each of their points they are tangent to the direction field at that point, and in that way the general qualitative nature of the solutions can be determined from the direction field. Direction fields can be drawn by hand for some simple ODEs, but Maple can draw them for any first-order equation. The Maple command to draw direction fields is *dfieldplot*, in the *DEtools* package [23-24].

3.5 Autonomous Equations

Equations of the form

$$\frac{dx}{dt} = f(x), \tag{12}$$

where the function f does not depend on t, are called autonomous equations. Such equations represent physical systems whose rules of evolution do not change over time and are particularly susceptible to qualitative analysis.

4 Materials and Methods

Problems involving ordinary differential equations are often difficult to analyze. Specifically, they are difficult to analyze based on the following criteria:

- Obtain the solution of an initial value problem in closed form.
- Describe the behavior of the solution (bounded or unbounded) on given intervals.
- Compare solutions to different initial value problems on given intervals, describing how changes in the initial value affect the solution.
- Graph, on the same interval, several solutions corresponding to different initial conditions.
- Explain the behavior of solution curves for large values of *x*.
- Identify the different types of solution curve behavior.

- Discuss the effect of small changes in initial values on the overall behavior of solution curves.
- Describe solution curves of implicitly given ordinary differential equations.
- Graph the solution curve of an initial value problem involving an implicitly given ordinary differential equation.
- Find, at a given point *x*, the value of the solution to an initial value problem involving an implicitly given ordinary differential equation.
- Study the continuous dependence of solutions to initial value problems on initial values.
- Graph direction fields in given two-dimensional domains.
- Compare the appearance of the direction field with respect to its solution curve.
- Describe equilibrium points across the direction field.

Therefore, the methodological part of this work aims to investigate and use various Maple commands that allow for the development of a complete analysis of ordinary differential equation problems, through numerical and graphical visualizations.

The methodological design includes the performance of numerical simulations in Maple, based on the construction and development of worksheets in Maple that contain the solution to the problems posed in the attached data recording sheet named Problem Set: Ordinary Differential Equations with Maple, which correspond to some exercises and problems contained in [22] and that have specifically been taken from [7] with the name *Problem Set B*: *First Order Equations*. The sheet contains problems concerning first-order ODEs. The design in Maple seeks to obtain graphic and numerical representations that help to carry out a convenient analysis and interpretation of the problems posed.

The developed methodology follows the following sequence:

a) Statement of the problem.
b) Mathematical analysis of the problem.
c) Solution in a Maple worksheet of the problem algebraically, numerically and graphically.
d) Exploration, description, analysis and interpretation of the model through the Maple interface.

4.1 Problem Set: Ordinary Differential Equations with Maple

1. The initial value problem $x\dfrac{dy}{dx}+2y=\sin x, \quad y(\pi/2)=1$

has the solution

$$y(x)=x^{-2}\left(\frac{\pi^2}{4}-1-x\cos x+\sin x\right).$$

Define this function in Maple and then:

a) Make a list of the values of $y(x)$ at $x=0.5,\ 1,\ 1.5,\ 2,\ \ldots,\ 5.$
b) Graph $y(x)$ on the intervals $0<x\le 2,\ 1\le x\le 10,$ and $10\le x\le 100.$ Describe the behavior of the solution near $x=0$ and for large values of $x.$
c) Find the solutions $y_j(x)$ of the differential equation corresponding to the initial conditions $y_j(\pi/2)=0.2j,\ j=1,\ldots,5.$

   Plot the functions $y_j(x),\ j=1,\ldots,5,$ on the same graph.
d) What do the solutions have in common near $x=0$? for large values of x?

   Can you find a solution of the differential equation that has no singularity at $x=0$? If so, graph it.

2. Consider the initial value problem

$$\begin{cases} x\dfrac{dy}{dx}+y=2x \\ y(1)=c \end{cases}$$

a) Solve it using Maple.
b) Use Maple to graph the solutions for $c=0.8,\ 0.9,\ 1,\ 1.1,\ 1.2$ on the interval $(0.75,\ 1.25)$.

c) Evaluate these solutions at $x = 0.01,\ 0.1,\ 1,\ 10.$

d) Now plot all five solutions together on the interval $(0,\ 2.5)$. How do changes in the initial data affect the solution as $x \to \infty$?, as $x \to 0^+$?

3. Solve the initial value problem:

$$\begin{cases} \dfrac{dy}{dx} - y = \cos x \\ \quad y(0) = c \end{cases}$$

Use Maple to graph solutions for $c = -0.9,\ \text{-}0.8, \ldots,\ \text{-}0.1,\ 0.$ Display all the solutions on the same interval between $x = 0$ and an appropriately chosen right endpoint. Explain what happens to the solution curves for large values of x. Identify three distinct types of behavior. Now discuss what effect small changes in initial data can have on the global behavior of solution curves.

4. Consider the differential equation $\dfrac{dy}{dx} = \dfrac{x - e^{-x}}{y + e^{y}}.$

a) Solve it using Maple. Observe that the solution is given implicitly in the form $f(x, y) = c.$

b) Use *contourplot* to see what the solution curves look like. For your x and y ranges, you might use $x = -1 \ldots 3$ and $y = -2 \ldots 2.$

c) Use *implicitplot* to plot the solution satisfying the initial condition $y(1.5) = 0.5.$ Your plot should show two curves. Indicate which one corresponds to the solution.

d) Given a value x_1 to find $y(x_1)$ you need to find the solution of $f(x_1, y) = f(1.5,\ 0.5)$ (viewed as an equation in y) near $y = 0.5.$ This can be done with the *fsolve* command. Find $y(0),\ y(1),\ y(1.8),\ y(2.1).$ Mark these values on your plot.

5. Consider the differential equation

$$(e^x \sin y - 2y \sin x)\ dx + (e^x \cos y + 2\cos x)\ dy = 0.$$

a) Solve it using Maple. Observe that the solution is given implicitly in the form $f(x, y) = c.$

b) Use *contourplot* to see what the solution curves look like. For your x and y ranges, you might use $x = -3 \ldots 3$ and $y = -3 \ldots 3.$

c) Use *implicitplot* to plot the solution satisfying the initial condition $y(0) = 0.5.$ Your plot should show several curves. Indicate which one corresponds to the solution.

d) Given a value x_1, to find $y(x_1)$ you need to find the solution of

$$f(x_1, y) = f(0,\ 0.5)$$

(viewed as an equation in y) near $y = 0.5.$ This can be done with the *fsolve* command. Find $y(-1),\ y(1),\ y(2).$ Mark these values on your plot.

6. In this problem we study continuous dependence of solutions on initial data.

a) Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{y}{1 + x^2}, \quad y(0) = c.$

b) Let y_c denote the solution in part (a). Use Maple to plot the solutions y_c for $c = -10,\ \text{-}9, \ldots,\ \text{-}1,\ 0,\ 1$ on one graph. Display all the solutions on the interval $-20 \le x \le 20.$

c) Compute $\lim\limits_{x \to \pm\infty} y_1(x).$

d) Now find a single constant L such that for all real x, we have

$$\left| y_a(x) - y_b(x) \right| \le L \left| a - b \right|,$$

for any pair of numbers a and b. Show that $L=1$ will work if we consider only negative values of x.

7. Consider the equation

$$\frac{dx}{dt}=e^{-t}-2x.$$

Plot the direction field on the rectangle $-2\le t\le 3,\ -1\le x\le 2.$ Find the general solution.

8. Consider the equation

$$\frac{dx}{dt}=x^2+t.$$

Obtain its direction field. Then discuss what happen with solutions.

9. Consider the equation

$$\frac{dx}{dt}=x^2-x.$$

Obtain the appearance of its direction field. Derive an explicit formula solution and draw the actual solution curves for the differential equation.

10. For the equation below, plot the direction field on a rectangle large enough (but not too large) to show clearly all of its equilibrium points.

$$\frac{dy}{dx}=-\frac{y(y-2)(y-4)}{10}.$$

11. For the equation below, plot the direction field on a rectangle large enough (but not too large) to show clearly all of its equilibrium points.

$$\frac{dy}{dx}=y^2-3y+1.$$

12. For the equation below, plot the direction field on a rectangle large enough (but not too large) to show clearly all of its equilibrium points.

$$\frac{dy}{dx}=0.1y-\sin y.$$

5 Results and discussion

The results of the Maple worksheets corresponding to the solution of *Problem Set. Ordinary Differential Equations with Maple* are presented. A discussion is held for each problem.

5.1 Finding the solution to the initial value problem:

$$x\frac{dy}{dx}+2y=\sin x,\qquad y(\pi/2)=1$$

> $sol1 := dsolve\left(\left\{x\cdot diff(y(x),x)+2\cdot y(x)=\sin(x), y\left(\frac{\pi}{2}\right)=1\right\}, y(x)\right);$

$$sol1 := y(x)=\frac{\sin(x)-\cos(x)\,x+\frac{1}{4}\pi^2-1}{x^2}$$

Defining this function in Maple:

> $y1 := unapply(rhs(sol1), x);$

$$y1 := x \rightarrow \frac{\sin(x) - \cos(x)\,x + \frac{1}{4}\pi^2 - 1}{x^2}$$

Making a list of the values of $y(x)$ in $x = 0.5,\ 1,\ 1.5,\ 2,\ \ldots,\ 5.$

> $evalf\left(\left\{y1\left(\frac{c}{2}\right)\$\ c = 1\,..10\right\}\right);$

{-0.03639336405, 0.07103467055, 0.2078233180, 0.3587115330,
0.5087220665, 0.6509971655, 0.8022480500, 1.048351238,
1.768569780, 6.03214143}

Fig. 1 shows the solution $y(x)$ on the interval $0 < x \leq 2,$ with which it can be stated that the solution close to $x = 0$ is unbounded and tends to $+\infty$.

>$plot(y1(x), x = 0\,..2);$

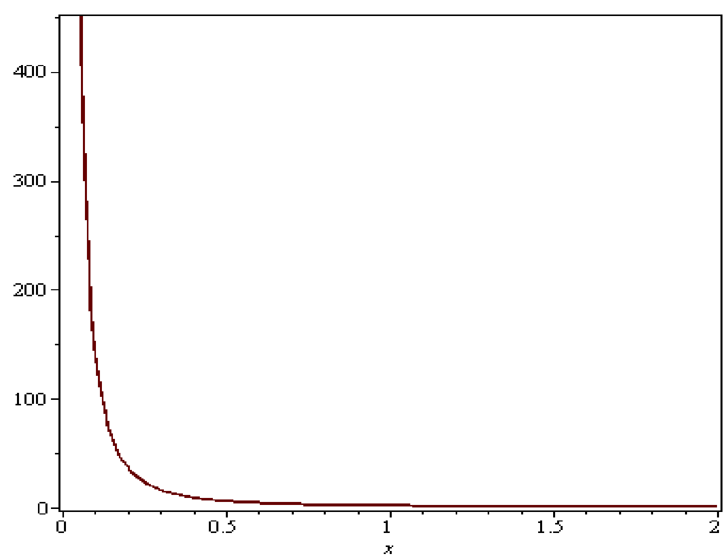

Figure 1: Solution to Problem 1(b) on the interval $[0, 2]$

Fig. 2 shows the solution $y(x)$ on the interval $1 \leq x \leq 10$.

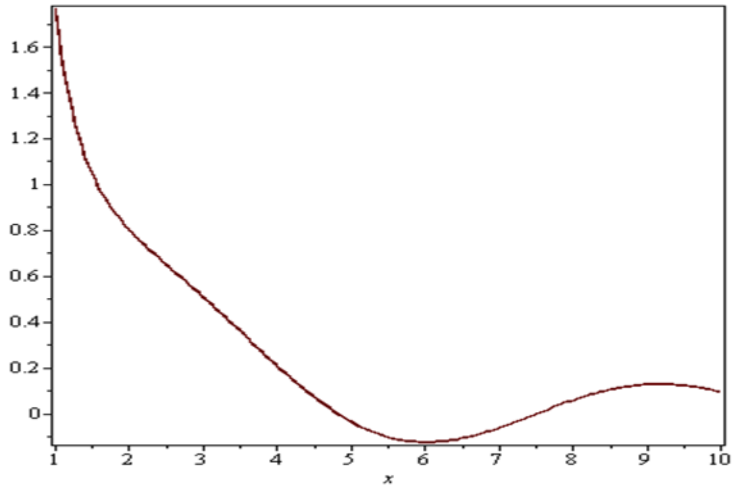

Figure 2: Solution to Problem 1(b) on the interval $[1,10]$

Fig. 3 shows the solution $y(x)$ on the interval $10 \le x \le 100$, with which it can be stated that the solution for large values of x tends to 0.

> $plot(y1(x), x = 10 ..100);$

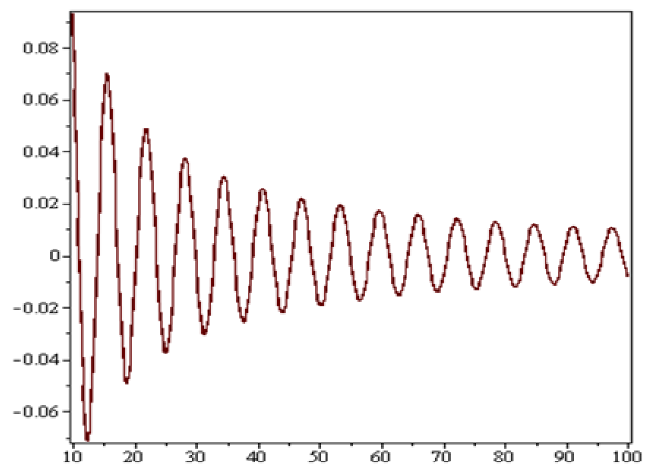

Figure 3: Solution to Problem 1(b) on the interval $[10,100]$

Finding the solutions $y_j(x)$ of the differential equation corresponding to the initial conditions

$$y_j(\pi/2) = 0.2\,j, \quad j = 1,\ldots,5:$$

>
$sol11 := dsolve\left(\left\{x \cdot diff(y(x), x) + 2 \cdot y(x) = \sin(x), y\left(\frac{\pi}{2}\right) = a\right\},\right.$
$\left. y(x)\right);$

$$sol11 := y(x) = \frac{\sin(x) - \cos(x)\,x + \frac{1}{4}\,a\,\pi^2 - 1}{x^2}$$

> $y11 := unapply(rhs(sol11), x, a);$

$$y11 := (x, a) \rightarrow \frac{\sin(x) - \cos(x)\,x + \frac{1}{4}\,a\,\pi^2 - 1}{x^2}$$

Fig. 4 shows, in the same graph, the solutions $y_j(x),\ j = 1, \ldots, 5,$ on the interval $[0,10]$.

> $plot(\{y11(x, 0.2 \cdot j)\$\ j = 1..5\}, x = 0.2..10);$

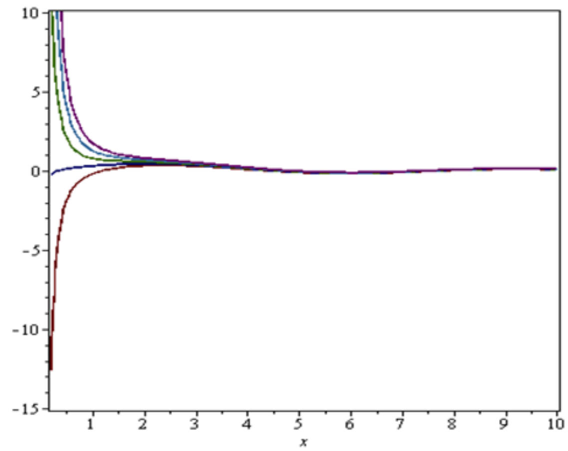

Figure 4: Solution to Problem 1(b) on the interval $[0,10]$

Fig. 5 shows, in the same graph, the solutions $y_j(x),\ j = 1, \ldots, 5,$ on the interval $[10,100]$.

> $plot(\{y11(x, 0.2 \cdot j)\$\ j = 1..5\}, x = 10..100);$

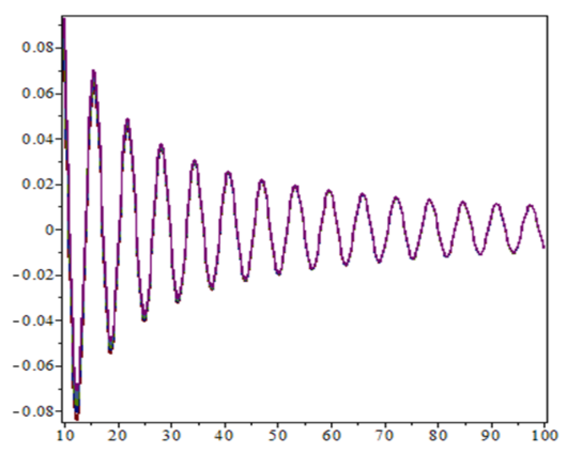

Figure 5: Solution to Problem 1(b) on the interval $[10,100]$

According to the graphs, it can be stated that the solutions, except one, near $x = 0$, are unbounded and tend to $+\infty$ and to $-\infty$; however, when $x \to +\infty$ they approach zero. Fig. 6 shows a solution, with its graph, of the ordinary differential equation without singularity in $x = 0$.

> $sol12 := dsolve(\{x \cdot diff(y(x), x) + 2 \cdot y(x) = \sin(x), y(0) = 0\}, y(x));$

$$sol12 := y(x) = \frac{\sin(x) - \cos(x)\, x}{x^2}$$

> $y12 := unapply(rhs(sol12), x);$

$$y12 := x \to \frac{\sin(x) - \cos(x)\, x}{x^2}$$

> $plot(y12(x), x = 0..10);$

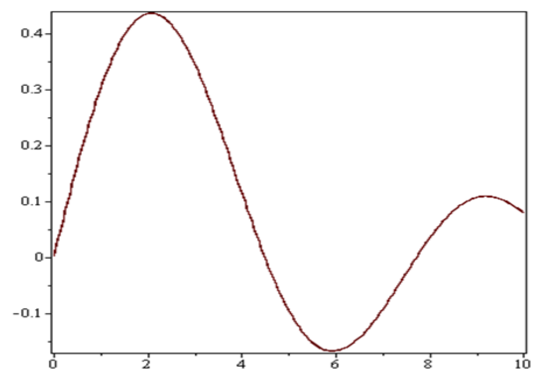

Figure 6: Solution to Problem 1(e) on the interval $[0,10]$, without singularity at $x = 0$

5.2 Finding the solution to the initial value problem:

$$\begin{cases} x\dfrac{dy}{dx} + y = 2x \\ y(1) = c \end{cases}$$

>$sol2 := dsolve(\{x \cdot diff(y(x), x) + y(x) = 2 \cdot x, y(1) = c\}, y(x));$

$$sol2 := y(x) = x + \frac{c - 1}{x}$$

>$y2 := unapply(rhs(sol2), x, c);$

$$y2 := (x, c) \to x + \frac{c - 1}{x}$$

Fig. 7 shows on $(0.75,\ 1.25)$ solutions for $c = 0.8,\ 0.9,\ 1,\ 1.1,\ 1.2$.

> $plot(\{y2(x, 0.8), y2(x, 0.9), y2(x, 1), y2(x, 1.1), y2(x, 1.2)\}, x = 0.75$
$..1.25);$

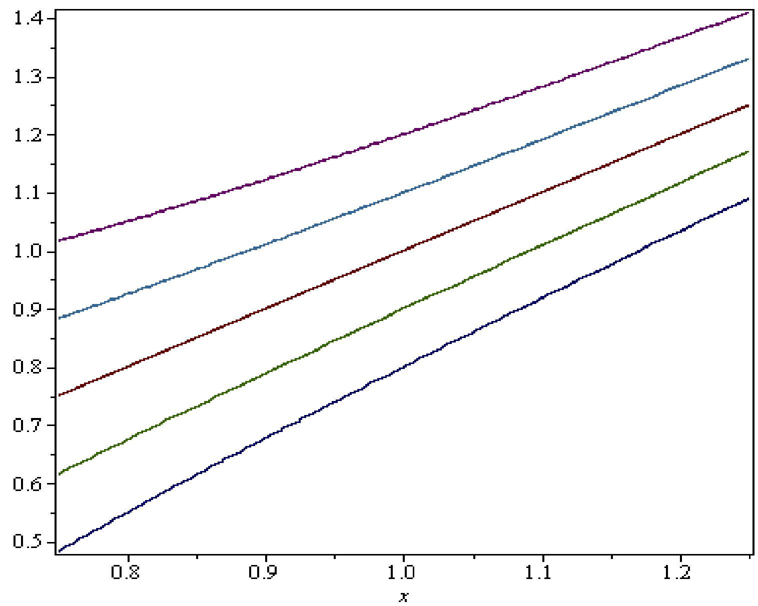

Figure 7: Solution to Problem 2(b) on the interval $(0.75,\ 1.25)$

These solutions are evaluated in $x = 0.01,\ 0.1,\ 1,\ 10$.

> $evalf(\{y2(0.01, 0.8), y2(0.01, 0.9), y2(0.01, 1), y2(0.01, 1.1), y2(0.01, 1.2)\});$

$\{-19.99, -9.99, 0.01, 10.01, 20.01\}$

> $evalf(\{y2(0.1, 0.8), y2(0.1, 0.9), y2(0.1, 1), y2(0.1, 1.1), y2(0.1, 1.2)\});$

$\{-1.9, -0.9, 0.1, 1.1, 2.1\}$

>$evalf(\{y2(1, 0.8), y2(1, 0.9), y2(1, 1), y2(1, 1.1), y2(1, 1.2)\});$
$\{0.8, 0.9, 1., 1.1, 1.2\}$

>$evalf(\{y2(10, 0.8), y2(10, 0.9), y2(10, 1), y2(10, 1.1), y2(10, 1.2)\});$
$\{9.980000000, 9.990000000, 10., 10.01000000, 10.02000000\}$

Fig. 8 and Fig. 9 show, in the same graph, the five solutions on the intervals $[0,\ 2.5]$ and $[100,\ 500]$, respectively.

> $plot(\{y2(x, 0.8), y2(x, 0.9), y2(x, 1), y2(x, 1.1), y2(x, 1.2)\}, x = 0..2.5);$

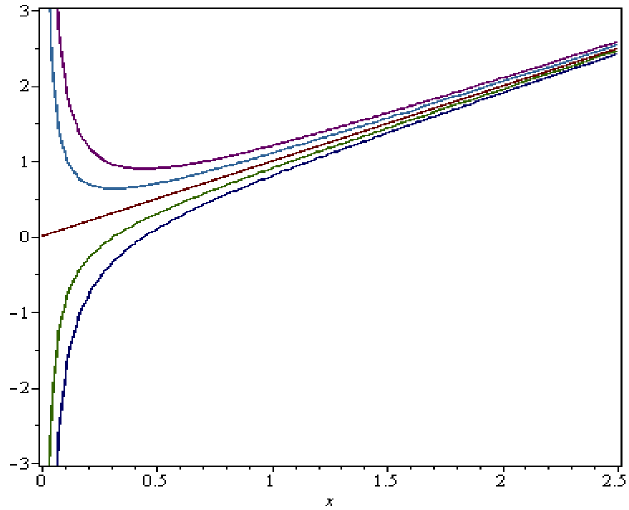

Figure 8: Solution to Problem 2(d) on the interval $[0,\ 2.5]$

> $plot(\{y2(x, 0.8), y2(x, 0.9), y2(x, 1), y2(x, 1.1), y2(x, 1.2)\}, x = 100..500);$

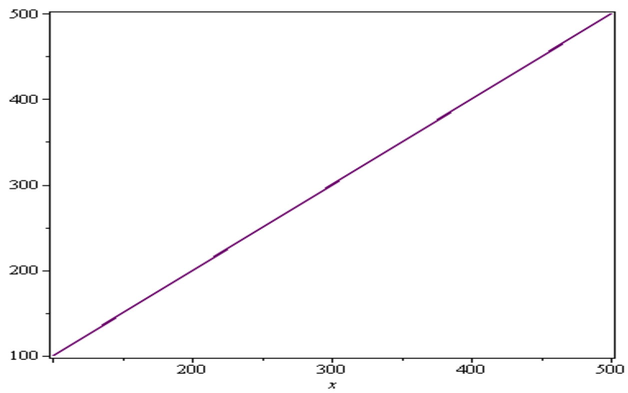

Figure 9: Solution to Problem 2(d) on the interval $[100,\ 500]$

Changes in the initial data affect the solution for $x \to 0^+$, but do not affect for $x \to +\infty$. The five solutions, except one, near $x = 0$ are unbounded and tend to $+\infty$ and to $-\infty$; however when $x \to +\infty$ they approach the exception: a straight line.

5.3 Finding the solution to the initial value problem:

$$\begin{cases} \dfrac{dy}{dx} - y = \cos x \\ y(0) = c \end{cases}$$

> $sol3 := dsolve(\{diff(y(x), x) - y(x) = \cos(x), y(0) = c\}, y(x));$

$$sol3 := y(x) = -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) + e^x\left(c + \frac{1}{2}\right)$$

$>y3 := unapply\left(rhs(sol3), x,c\right);$

$$y3 := (x, c) \rightarrow -\frac{1}{2}\cos(x) + \frac{1}{2}\sin(x) + e^{x}\left(c + \frac{1}{2}\right)$$

The solutions are plotted in Fig. 10 for $c = -0.9, -0.8, \ldots, -0.1, 0.$ The solutions are presented on the same interval between $x = 0$ and an appropriate right endpoint $x = 4$.

$> plot\left(\left\{y3(x, -0.9), y3(x,-0.8), y3(x,-0.7), y3(x,-0.6), y3(x,-0.5), y3(x,-0.4), y3(x,-0.3), y3(x,-0.2), y3(x,-0.1), y3(x, 0)\right\}, x = 0..4\right)$
;

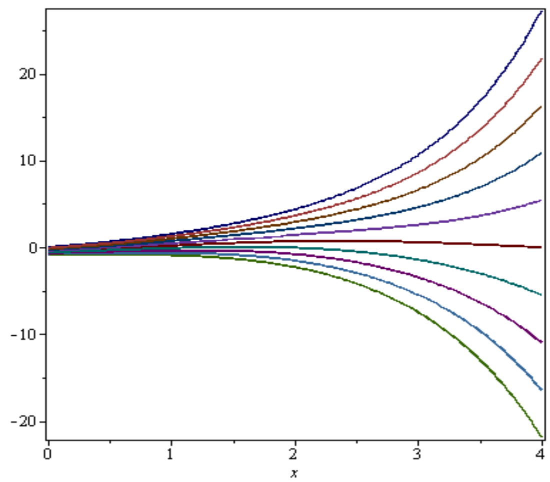

Figure 10: Solution to Problem 3 on the interval $[0,\ 4]$

The aim is to explain the behavior of the solution curves for large values of x. Likewise, discuss the effect that small changes in the initial data can have on the global behavior of the solution curves.

$>plot\left(\left\{y3(x, -0.9), y3(x,-0.8), y3(x,-0.7), y3(x,-0.6)\right\}, x = 0..10\right);$

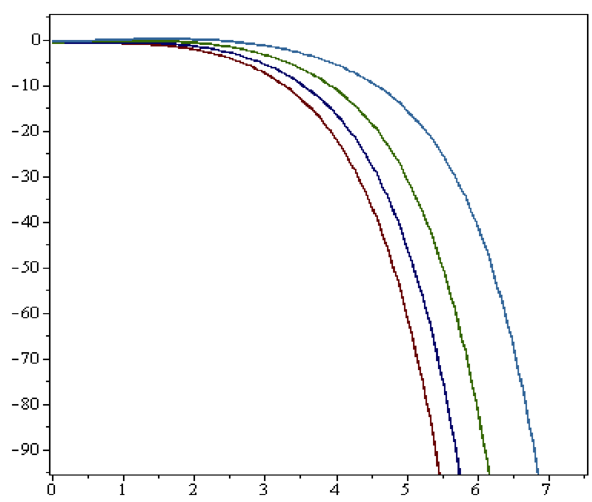

Figure 11: Solution curves (first type) of Problem 3 on the interval $[0,\ 10]$

>$plot(y3(x,-0.5), x = 0..10);$

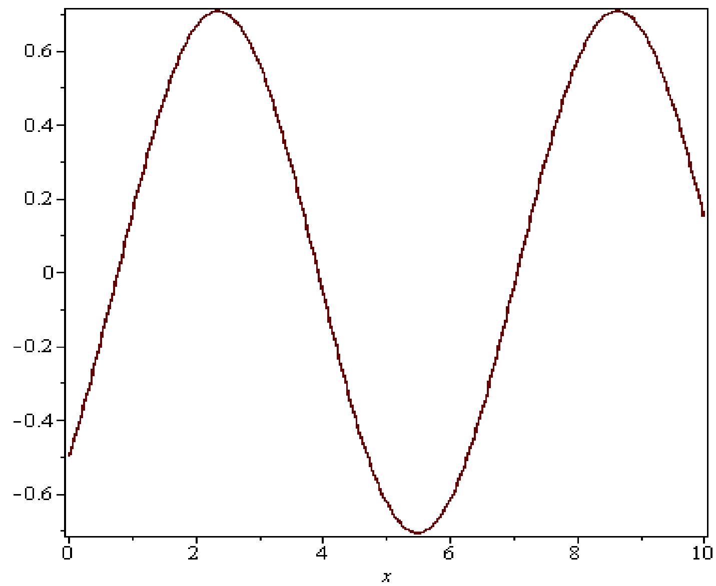

Figure 12: Solution curve (second type) of Problem 3 on the interval $[0, 10]$

> $plot(\{ y3(x,-0.4), y3(x,-0.3), y3(x,-0.2), y3(x,-0.1), y3(x, 0) \}, x = 0$
$..10);$

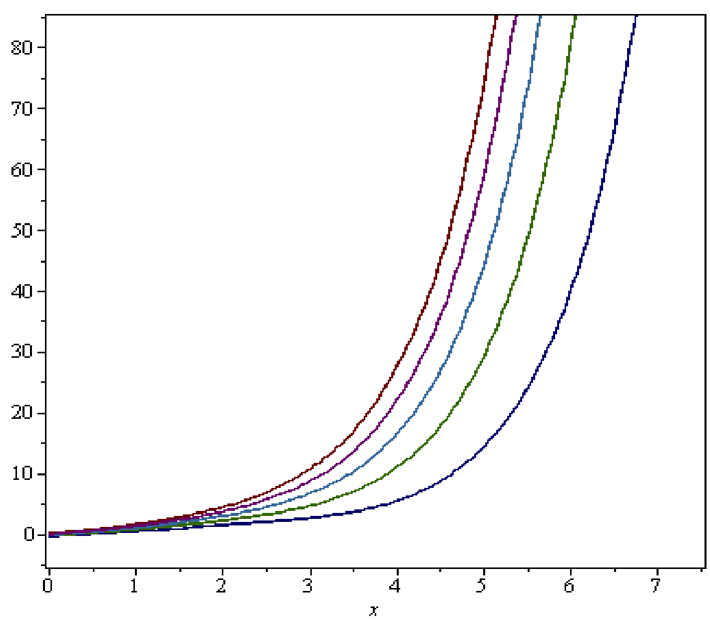

Figure 13: Solution curves (third type) of Problem 3 on the interval $[0, 10]$

According to Fig. 11, Fig. 12 and Fig. 13, three types of behavior are identified for the solution curves. Small changes in the initial data do not affect the solution curves for values greater than and close to $x = 0$; however, for large positive values of x, curves of the first type tend to $-\infty$, those of the second type are bounded and tend to 0, while those of the third type tend to $+\infty$.

5.4 Finding the implicit solution to the ordinary differential equation:

$$\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^{y}}.$$

$> sol4 := dsolve\left(diff(y(x), x) = \dfrac{x - \exp(-x)}{y(x) + \exp(y(x))}, y(x) \right);$

$sol4 := \frac{1}{2} x^2 + \mathrm{e}^{-x} - \frac{1}{2} y(x)^2 - \mathrm{e}^{y(x)} + \_C1 = 0$

$> f4 := (x, y) \rightarrow \frac{1}{2} x^2 + \mathrm{e}^{-x} - \frac{1}{2} y^2 - \mathrm{e}^{y};$

$f4 := (x, y) \rightarrow \frac{1}{2} x^2 + \mathrm{e}^{-x} - \frac{1}{2} y^2 - \mathrm{e}^{y}$

It is noted that the solution is implicitly given in the form $f(x, y) = c$.
In Fig. 14, *contourplot* has been used to see what the solution curves look like. For the ranges of x and y, it has been taken $x = -1 \ldots 3$ and $y = -2 \ldots 2$.

$> with(plots):$
$> contourplot(f4(x, y), x = -1..3, y = -2..2);$

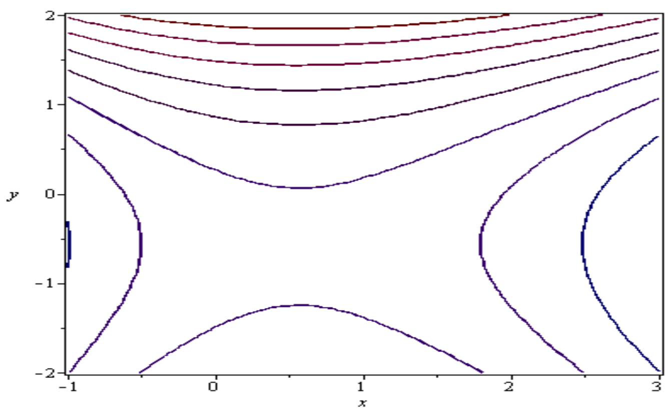

Figure 14: Solution curves of Problem 4(b) with $-1 \le x \le 3$, $-2 \le y \le 2$, first form

Fig. 15 presents a second way to use *contourplot* to see what the solution curves look like. In Fig. 16, *implicitplot* has been used to graph the solution satisfying the initial condition $y(1.5) = 0.5$. The graph shows two curves, one of them is the solution, which is showed in Fig. 17.

$> d := f4(1.5, 0.5);$
$d := -0.425591111$
$> g4 := (x, y) \rightarrow \frac{1}{2} x^2 + \mathrm{e}^{-x} - \frac{1}{2} y^2 - \mathrm{e}^{y} - d;$

$g4 := (x, y) \rightarrow \frac{1}{2} x^2 + \mathrm{e}^{-x} - \frac{1}{2} y^2 - \mathrm{e}^{y} - d$
$> g4(1.5, 0.5);$
$0.$
$> contourplot(g4(x, y), x = -1..3, y = -2..2);$

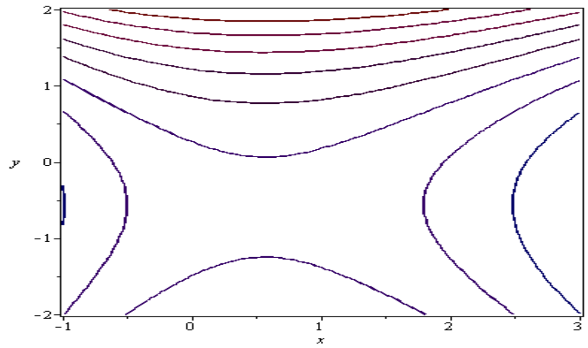

Figure 15: Solution curves of Problem 4(b) with $-1 \le x \le 3$, second form

$>$*implicitplot*$(f4(x, y) = f4(1.5, 0.5), x = -1..3, y = -2..2);$

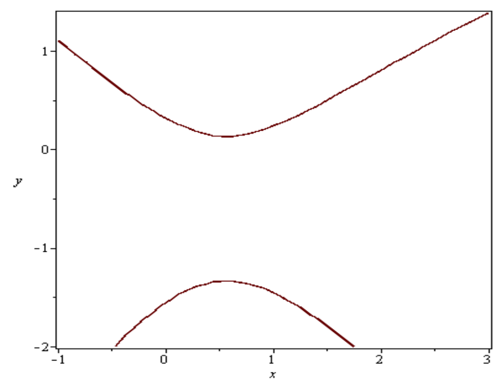

Figure 16: Probable solution curves of Problem 4(c) with $-1 \le x \le 3$

$>$*implicitplot*$(f4(x, y) = f4(1.5, 0.5), x = -1..3, y = 0..2);$

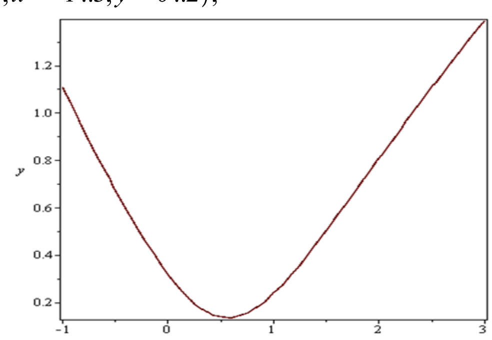

Figure 17: Solution curve of Problem 4(c) with $-1 \le x \le 3$, $0 \le y \le 2$

Given a value x_1 to find $y(x_1)$ it is necessary to find the solution of $f(x_1, y) = f(1.5,\ 0.5)$ (viewed as an equation in y) near of $y = 0.5$. This can be done with the *fsolve* command. It's found $y(0)$, $y(1)$, $y(1.8)$, $y(2.1)$. In Fig. 18 these values are marked on a graph.

$>$ *fsolve*$(f4(0, y) = f4(1.5, 0.5), y);$
0.3183856549
$>$*fsolve*$(f4(1, y) = f4(1.5, 0.5), y);$
0.2356325919

> $fsolve(f4(1.8, y) = f4(1.5, 0.5), y);$

0.6821874213

> $fsolve(f4(2.1, y) = f4(1.5, 0.5), y);$

0.8662118724

$PLOT(\mathrm{POINTS}([[0, fsolve(f4(0, y) = f4(1.5, 0.5), y)], [1, fsolve(f4(1, y) = f4(1.5, 0.5), y)], [1.8, fsolve(f4(1.8, y) = f4(1.5, 0.5), y)], [2.1, fsolve(f4(2.1, y) = f4(1.5, 0.5), y)]], \mathrm{SYMBOL}(\mathrm{DIAMOND})));$

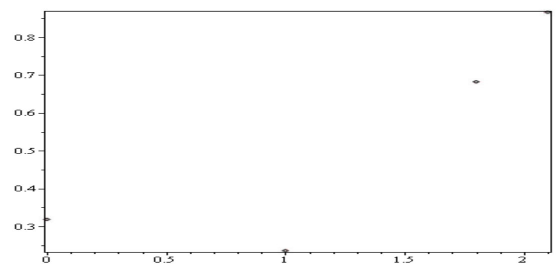

Figure 18: Some points of the solution curve of Problem 4(d)

5.5 Finding the implicit solution to the ordinary differential equation:

$$(e^x \sin y - 2y \sin x)\ dx + (e^x \cos y + 2\cos x)\ dy = 0.$$

> $sol5 := dsolve\left(diff(y(x), x) = \dfrac{2 \cdot y(x) \cdot \sin(x) - \exp(x) \cdot \sin(y(x))}{\exp(x) \cdot \cos(y(x)) + 2 \cdot \cos(x)}, y(x) \right);$

$sol5 := 2\, y(x) \cos(x) + \mathrm{e}^x \sin(y(x)) + \_C1 = 0$

> $f5 := (x, y) \rightarrow 2 \cdot y \cdot \cos(x) + \mathrm{e}^x \cdot \sin(y);$

$f5 := (x, y) \rightarrow 2\, y \cos(x) + \mathrm{e}^x \sin(y)$

It is noted that the solution is implicitly given in the form $f(x, y) = c$. In Fig. 19, *contourplot* has been used to see what the solution curves look like. For the ranges of x and y, it has been taken $x = -3 \ldots 3$ and $y = -3 \ldots 3$.

> $with(plots):$

> $contourplot(f5(x, y), x = -3\,..3, y = -3\,..3);$

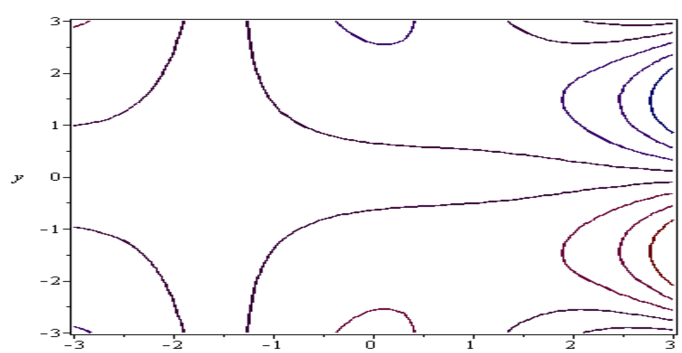

Figure 19: Solution curves of Problem 5(b) with $-3 \le x \le 3$, $-3 \le y \le 3$

In Fig. 20, *implicitplot* has been used to graph the solution satisfying the initial condition $y(0)=0.5$. The graph shows several curves, one of them is the solution, given in Fig. 21.

>$implicitplot(f5(x,y)=f5(0,0.5), x=-3..3, y=-3..3);$

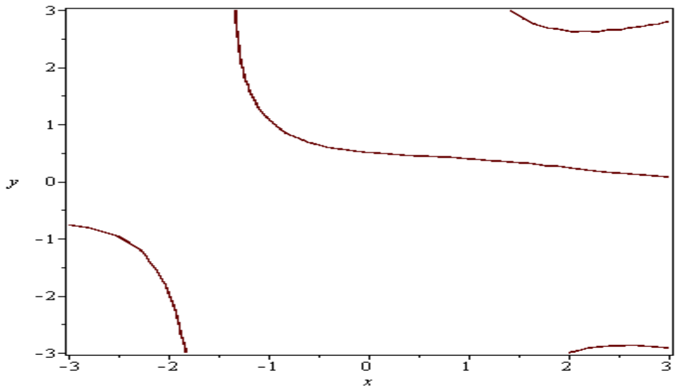

Figure 20: Probable solution curves of Problem 5(c) with $-3 \le x \le 3$, $-3 \le y \le 3$

>$implicitplot(f5(x,y)=f5(0,0.5), x=-1.8..2, y=-3..2.5);$

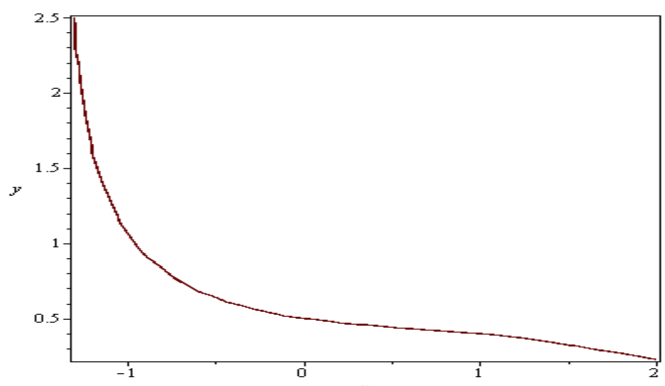

Figure 21: Solution curve of Problem 5(c) with $-1.8 \le x \le 2$, $0 \le y \le 2.5$

Given a value x_1, to find $y(x_1)$ it is necessary to find the solution of $f(x_1,y)=f(0,\ 0.5)$ (viewed as an equation in y) near $y=0.5$. This can be done with the *fsolve* command. It's found $y(-1),\ y(1),\ y(2)$. In Fig. 22 these values are marked on a graph.

>$fsolve(f5(-1,y)=f5(0,0.5), y);$
1.070377493
>$fsolve(f5(1,y)=f5(0,0.5), y);$
0.3968306836
>$fsolve(f5(2,y)=f5(0,0.5), y);$
0.2278495202

> $PLOT(\mathrm{POINTS}([[-1, fsolve(f5(-1,y)=f5(0,0.5),y)], [1, fsolve(f5(1,y)=f5(0,0.5),y)], [2, fsolve(f5(2,y)=f5(0,0.5), y)]], \mathrm{SYMBOL}(\mathrm{DIAMOND})));$

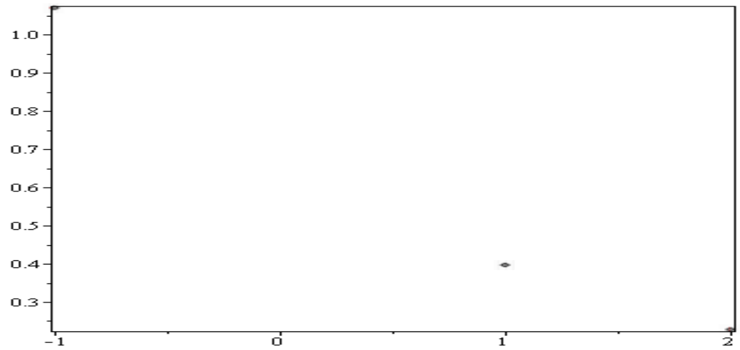

Figure 22: Some points of the solution curve of Problem 5(d)

5.6 In this problem the continuous dependence of the solutions with respect to the initial data is studied. Finding the solution to the initial value problem:

$$\frac{dy}{dx} = \frac{y}{1+x^2}, \quad y(0) = c.$$

$>sol6 := dsolve\left(\left\{diff(y(x), x) = \frac{y(x)}{1+x^2}, y(0) = c\right\}, y(x)\right);$

$sol6 := y(x) = c\,e^{\arctan(x)}$

$>y6 := unapply(rhs(sol6), x, c);$

$y6 := (x, c) \rightarrow c\,e^{\arctan(x)}$

It is denoted as y_c the solution of the given problem. In Fig. 23, for $c = -10,\ \text{-}9, \ldots,\ \text{-}1,\ 0,\ 1,$ the solutions y_c are graphed together on the interval $-20 \le x \le 20$. Fig. 24 shows the solution curve on the interval $[-30,\ 100]$.

$>plot(\{y6(x, c)\$\ c = -10\,..10\}, x = -20\,..20);$

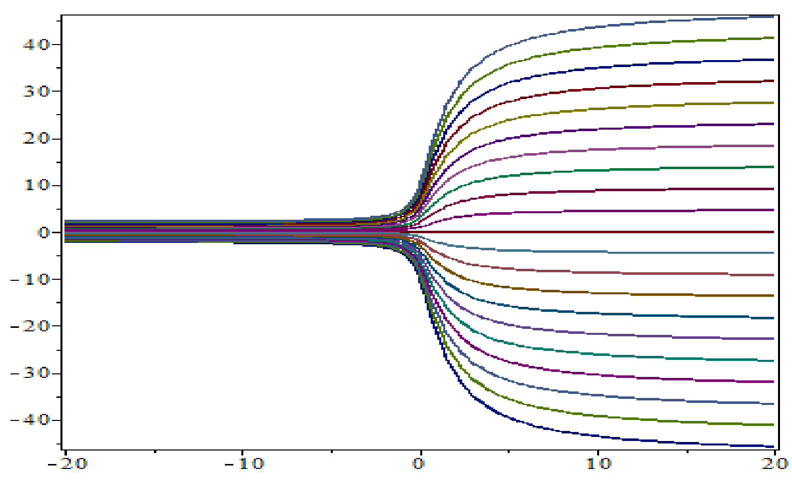

Figure 23: Solution curves of Problem 6(b) on the interval $[-20,\ 20]$

$>plot(y6(x,1), x=-30..100);$

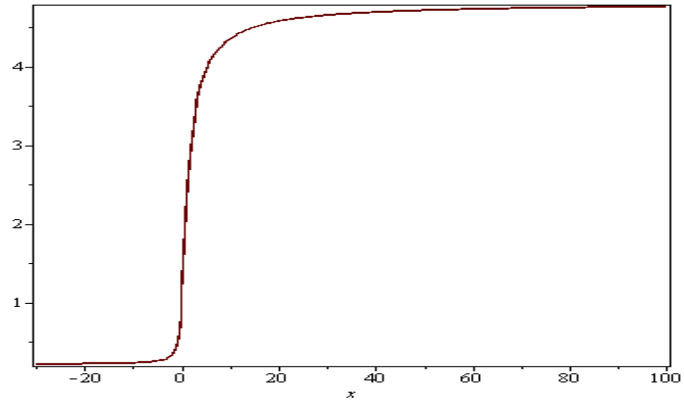

Figure 24: Solution curve y_1 of Problem 6(b) on the interval $[-30,\ 100]$

Computing $\lim\limits_{x\to\pm\infty} y_1(x)$:

$> \lim\limits_{x\to+\infty} y6(x,1);$

$e^{\frac{1}{2}\pi}$

$>evalf\left(\exp\left(\frac{\pi}{2}\right)\right);$

4.810477382

$> \lim\limits_{x\to-\infty} y6(x,1);$

$e^{-\frac{1}{2}\pi}$

$>evalf\left(\exp\left(-\frac{\pi}{2}\right)\right);$

0.2078795763

For any pair of numbers a, b and $x \in \mathbb{R}$:

$$\left|y_a(x) - y_b(x)\right| = \left|ae^{\tan^{-1}x} - be^{\tan^{-1}x}\right| = \left|(a-b)e^{\tan^{-1}x}\right| \le L\left|a-b\right|.$$

According to Fig. 25 it can be taken $L=5, \forall x \in \mathbb{R}$, and $L=1,\ \forall x<0$.

$>plot\left(\exp\left(\tan^{-1}(x)\right), x=-30..100\right);$

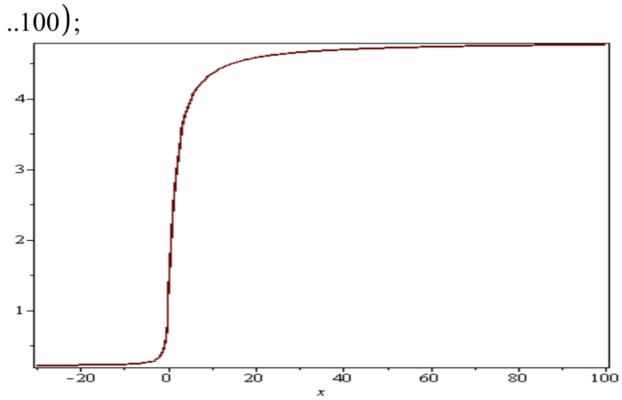

Figure 25: Graph of the function $e^{\tan^{-1}(x)}$ from Problem 6(d) on the interval $[-30,\ 100]$

5.7 Plotting the direction field of the next equation on the rectangle $-2 \le t \le 3,\ -1 \le x \le 2$, **and finding the general solution.**

$$\frac{dx}{dt} = e^{-t} - 2x.$$

$>with(DEtools):$

$dfieldplot(diff(x(t),t) = \exp(-t) - 2\cdot x(t), x(t), t=-2..3, x=-1..2, arrows = LINE, axes = BOXED, color = \exp(-t) - 2\cdot x(t))$

$>$

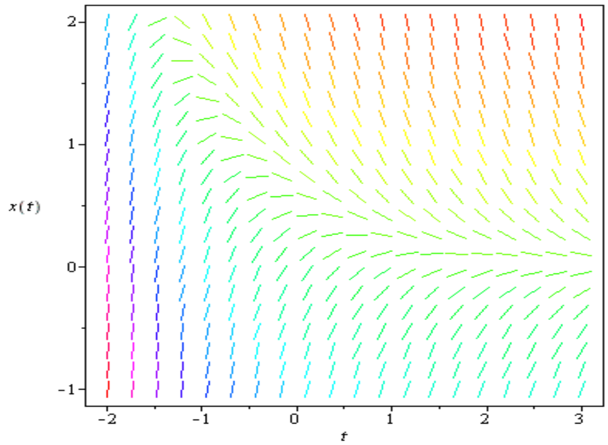

Figure 26: Direction field of the ordinary differential equation given on Problem 7

The direction field strongly suggests that if a solution is negative at some point, then it is increasing at that point, and that all solutions approach zero as soon as $t \to \infty$.The general solution of the differential equation is $x(t) = e^{-t} + ce^{-2t}$, which is obtained as:

$>dsolve(diff(x(t),t) = \exp(-t) - 2\cdot x(t), x(t));$

$x(t) = \left(e^{t} + \_C1\right) e^{-2\,t}$

$>y := (t,c) \rightarrow \exp(-t) + c\cdot\exp(-2\cdot t);$

$y := (t,c) \rightarrow e^{-t} + c\, e^{-2\,t}$

$>plot([y(t,-4), y(t,-3), y(t,-2), y(t,-1), y(t,0), y(t,1), y(t,2), y(t,3), y(t,4)], t=-2..3);$

Fig. 27 shows several of these solutions.

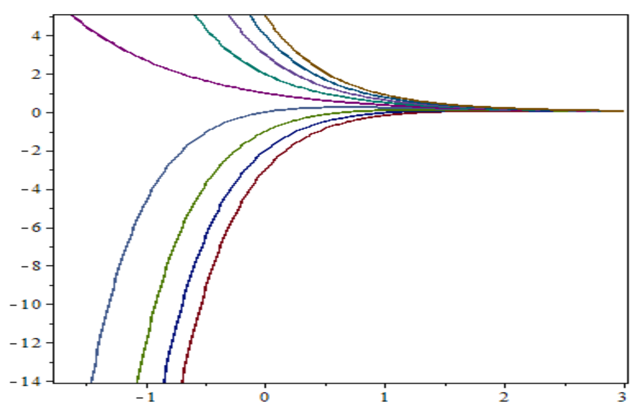

Figure 27: General solution of the ordinary differential equation given on Problem 7

5.8 Obtaining the direction field of the next equation and discussing what happens with solutions.

$$\frac{dx}{dt} = x^2 + t.$$

From the direction field obtained in Fig. 28, it seems that all the solutions are increasing for $t > 0$, and that all solutions eventually become positive. Fig. 28 also suggests that all solutions approach infinity. The question could be asked if this happens at a finite value of t or only at $t = \infty$. It is not easy to decide on the basis of Fig. 28, but in fact for each solution $x(t)$ there is a finite value t^* such that $\lim_{t \to t^*} x(t) = \infty$. This is a case of a differential equation that cannot be solved through an explicit formula. For these cases, a combination of qualitative and numerical tools is applied to analyze such equations, and in particular establish what is stated for the given differential equation.

$>with(DEtools):$

$dfieldplot\left(diff(x(t), t) = x(t)^2 + t, x(t), t = -2..2, x = -2..2, arrows = LINE, axes = BOXED, color = x(t)^2 + t\right);$

$>$

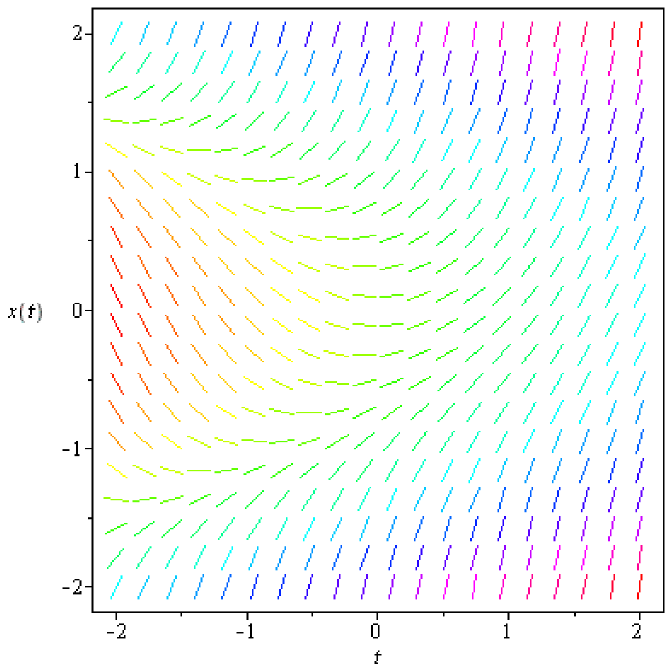

Figure 28: Direction field of the ordinary differential equation given on Problem 8

5.9 Obtaining the appearance of its direction field of the next equation and deriving an explicit formula solution.

$$\frac{dx}{dt} = x^2 - x.$$

$>with(DEtools):$

$dfieldplot\left(diff(x(t), t) = x(t)^2 - x(t), x(t), t = -1..4, x = -2..4, arrows = LINE, axes = BOXED, color = x(t)^2 - x(t)\right);$

$>$

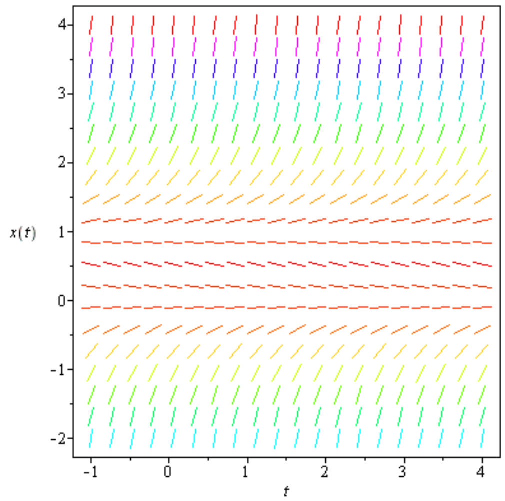

Figure 29: Direction field of the ordinary differential equation given on Problem 9

The general solution of the differential equation, with the initial condition $x(0)=x_0$, is:

$$x(t)=\frac{x_0}{(1-x_0)e^t+x_0}$$

which in Maple is obtained as:

$>dsolve\left(\{diff(x(t),t)=x(t)^2-x(t), x(0)=x0\}, x(t)\right);$

$$x(t)=-\frac{x0}{e^t\,x0-e^t-x0}$$

$>y := (t,c)\rightarrow\frac{c}{(1-c)\cdot\exp(t)+c};$

$$y:=(t,c)\rightarrow\frac{c}{(1-c)\,e^t+c}$$

$> plot([y(t,-1.8), y(t,-1.6), y(t,-1.4), y(t,-1.2), y(t,-1), y(t,-0.8), y(t,-0.6), y(t,-0.4), y(t,-0.2), y(t,0), y(t,0.2), y(t,0.4), y(t,0.6), y(t,0.8), y(t,1), y(t,1.2), y(t,1.4), y(t,1.6)], t=0..4, y=-2..4);$

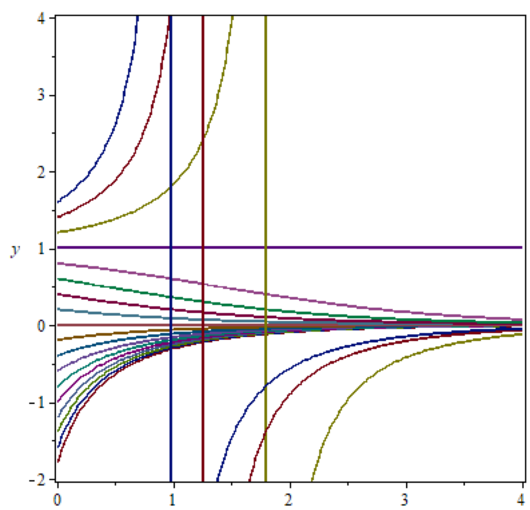

Figure 30: General solution of the ordinary differential equation given on Problem 9

5.10 Plotting the direction field of the next equation on a rectangle large enough (but not too large) to show clearly all of its equilibrium points.

$$y' = -y(y-2)(y-4)/10.$$

$$> \, dfieldplot\left(diff(y1(x), x) = -\frac{y1(x)\cdot(y1(x)-2)\cdot(y1(x)-4)}{10}, y1(x), x = -10..10, y1 = -8..8, arrows = LINE, axes = BOXED, color = -\frac{y1(x)\cdot(y1(x)-2)\cdot(y1(x)-4)}{10} \right);$$

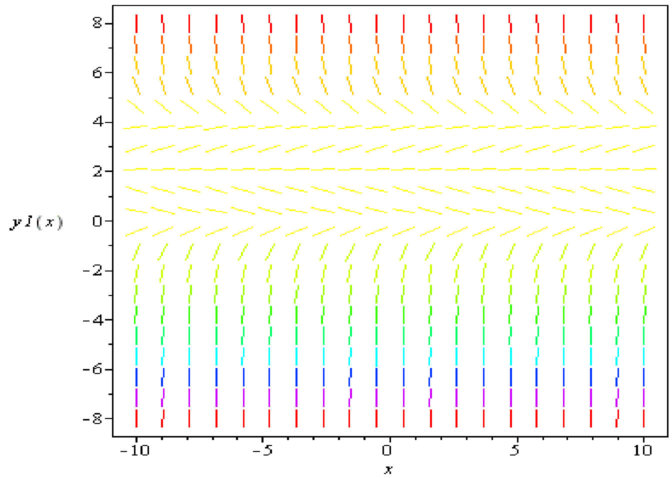

Figure 31: Direction field of the ordinary differential equation given on Problem 10

5.11 Plotting the direction field of the next equation on a rectangle large enough (but not too large) to show clearly all of its equilibrium points.

$$y' = y^2 - 3y + 1.$$

$$dfieldplot\left(diff(y2(x), x) = y2(x)^2 - 3\cdot y2(x) + 1, y2(x), x = -5..10, y2 = -3..7, arrows = LINE, axes = BOXED, color = y2(x)^2 - 3\cdot y2(x) + 1\right);$$

>

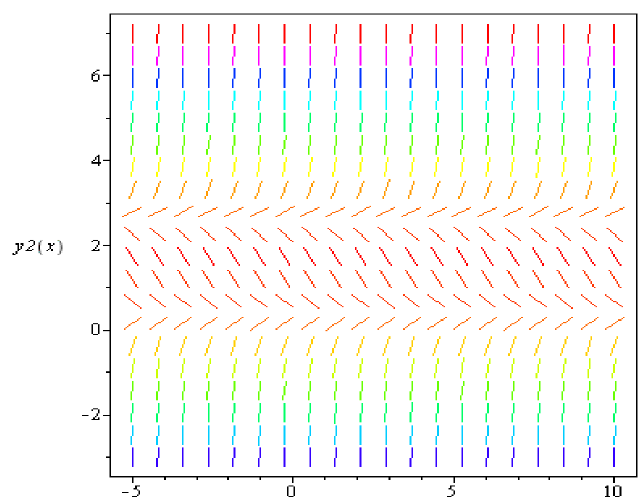

Figure 32: Direction field of the ordinary differential equation given on Problem 11

5.12 Plotting the direction field of the next equation on a rectangle large enough (but not too large) to show clearly all of its equilibrium points.

$$y' = 0.1y - \sin y.$$

> $dfieldplot(diff(y3(x), x) = 0.1 \cdot y3(x) - \sin(y3(x)), y3(x), x = -5..10, y3 = -3..7, arrows = LINE, axes = BOXED, color = 0.1 \cdot y3(x) - \sin(y3(x)));$

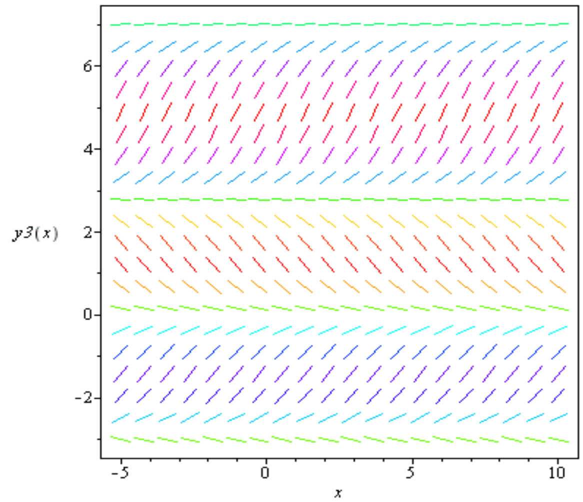

Figure 33: Direction field of the ordinary differential equation given on Problem 12

6 Conclusions

Certain Maple commands were identified and utilized that allowed for the effective development of mathematical analysis of ordinary differential equation problems, obtaining both numerical and graphical visualizations of the solutions. In general, this analysis allowed for obtaining the solution to an initial value problem in closed form, describing the behavior of the solution (bounded or unbounded) on certain intervals, comparing solutions to different initial value problems, graphing (on the same interval) several solutions corresponding to different initial conditions, identifying different types of behavior of solution curves, studying the continuous dependence of solutions to initial value problems on initial values, describing and graphing solution curves of implicitly given ordinary differential equations, graphing direction fields in certain two-dimensional domains by establishing comparisons with respect to their solution curve, as well as describing equilibrium points through the direction field.

In this work, the production of worksheets in Maple was achieved, which combine text with the numerical, symbolic and graphic output of the mathematical software.

Through Maple it has been possible to discuss the existence, uniqueness and stability of solutions of ODEs considered in the data record: Problem Set. Ordinary Differential Equations with Maple, topics that are fundamental in the theory and application of these equations. In each of the problems solved, it has been possible to explore, describe, analyze and interpret solutions of each equation, in view of Maple's simple and interactive interface.

Through Maple, it has been possible to implement quantitative and qualitative methods in first-order ODEs, visualized algebraically, numerically and graphically. In particular, a qualitative approach to the study of these equations has been considered. With this approach, qualitative information about the solutions is obtained directly from the equation, without the use of a formula for the solution.

Due to the great symbolic capacity of the software, it is concluded that Maple is very powerful for the quantitative and qualitative analysis of first-order ODEs. The Maple symbolic calculation software made it possible to efficiently carry out calculations and present solutions algebraically, numerically and graphically, so that the researcher can spend more time assimilating the concepts and analyzing the problems. It is concluded that the use of mathematical software allows the effective resolution of numerous problems formulated in terms of ODEs and constitutes a fundamental tool in scientific research.

Acknowledgements

The first author acknowledges the partial funding from the Peruvian state through the research bonus received at Universidad Nacional Pedro Ruiz Gallo, of Peru.

CRediT author statement.

Luis Jaime Collantes Santisteban: Conceptualization, Formal analysis, Software, Writing – original draft preparation. **Alberto Hananel**: Validation. **Samuel Collantes Santisteban**: Software, Writing – review and editing. **Rosa Gonzales**: Methodology, Translation.

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